

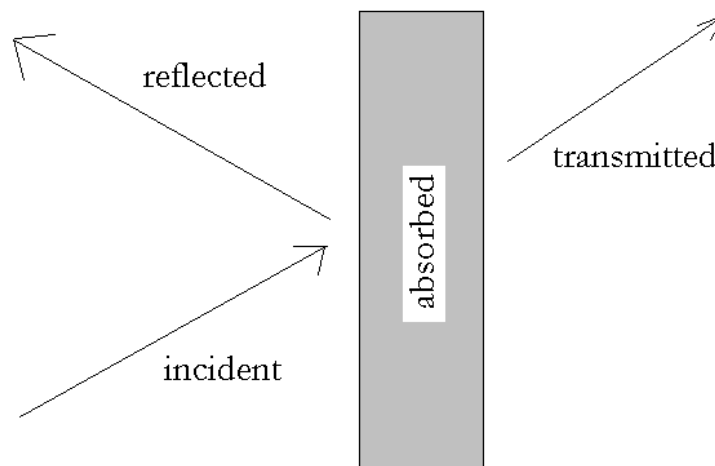
Chapter 10: Reverberation and room acoustics

In this chapter, we will explore physical room environments. Specifically, we will study how the walls of a room reflect and absorb sound. Sound reflections are important to consider; they help us localize the source of sounds and add richness to music. Anyone who has sung in the shower before knows that the unique acoustical environment a small, reflective environment provides can enhance a performance.

Reverberation is the phenomenon where sound waves persist in a room or space after the original source has ceased. I will quantify this phenomenon in terms of the decibel scale discussed in the last chapter. Of course, many factors come into play; room size, the materials our walls are made from, and the frequency of the sound must all be considered. Calculations we do here, though simplified, help us formally figure out why certain environments make good recording studios, others make good concert halls, and others are good for speeches.

What happens to sound waves in a room?

Acoustics considers more than the straight-line travel of sound from source to recipient. Room environments make a big difference in what listeners perceive. This is because sound can do a number of things when it contacts a wall, floor or ceiling.



An extremely simplified model of what an incident sound does at a wall.

By far, the most common thing a sound will do when it hits a wall is bounce off in some way. Though I've drawn the sound wave above reflecting at roughly the same angle at which it approached the wall, it is more complicated than that. The surface might be textured, which will result in a *scattering* of sound in multiple directions.

Large, stiff walls often reflect 90% or more of the energy that fall on them. We can control transmission through a wall by making the wall a different thickness or adding insulation.

Perception, echoes, and precedence

If you've been to the Grand Canyon (as I haven't), you are probably familiar with echoes. Even if you haven't been to the canyon, you've heard of echoes before; they are reflections that make it seem like a given sound is emitted more than once. These are certainly possible in a very reflective environment, but sometimes the reflections are much more subtle. In most small rooms, the effect of sound reflections is that a sound will seem to persist longer and "fill" the room better without the appearance of echoes.

If a sound and an echo arrive farther than about 50 milliseconds apart, you will likely not hear distinct echoes. Rather, the original sound will persist. The *precedence effect* is a phenomenon that helps you locate sounds. If you receive an original source, and a number of reflections arrive slightly later, the source you hear first will often determine the direction you localize the sound as originating from.

As stated above, *reverberation* is the phenomenon where sound waves persist in a room or space after the original source has ceased. I will now discuss ways we quantify reverberation. Then, we'll figure out how to calculate it.

Room modes

Just like one-dimensional strings admit certain frequencies that create interference-related feedback and standing waves, we can determine frequencies that would lead to standing waves in a room of given geometry. The problem is a little harder since the problem is three-dimensional, but hopefully one can at least see how the final formula is plausible.

Recall that for strings, the wavelength of a standing wave is twice the size of a node-node "loop." The size of a node-node gap for a string of length L is given by

$$NN = \frac{L}{n}$$

where n is a positive integer (1, 2, 3, etc.) That means the allowed wavelengths are

$$\lambda = \frac{2L}{n}$$

Since we know for a wave that $v = f\lambda$, we can solve for the allowed frequencies.

$$f = \frac{v}{\lambda} = \frac{v \times n}{2L}$$

This math applies to what we call the *axial* modes, that is, wave modes that reflect between two surfaces. However, other modes exist involving reflections from four or six surfaces (these are called

tangential and *oblique*, respectively). If we include all these possibilities, the allowed frequency formula gets more complicated. It has a part that looks like the Pythagorean theorem solved for a hypotenuse, and that is no accident. If L , W , and H are the room dimensions and n , m , and p are either zero or positive integers, the excitable frequencies for sound waves of speed v are given by

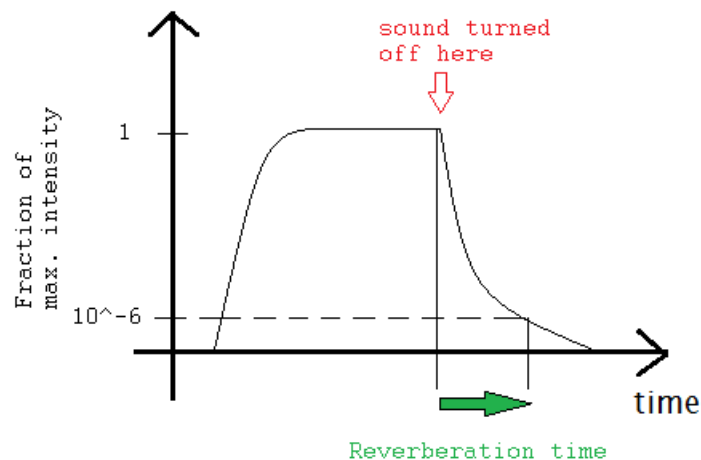
$$f = \frac{v}{2} \sqrt{\left(\frac{n}{L}\right)^2 + \left(\frac{m}{W}\right)^2 + \left(\frac{p}{H}\right)^2}$$

Again, the terms n , m , and p may take the values 0, 1, 2, and so on. Therefore, a number of frequencies allow for standing waves in a room. If two of these three wave numbers are zero, this equation reduces to the simple axial mode formula above.

Websites exist online that accept room dimensions as an input and produce a list of excitable modes as output. These calculators basically input a bunch of possibilities for the n , m , and p numbers and list the results. See [here](#). Of course, if your room is not a simple rectangular one, the math gets more complicated still.

Reverberation time

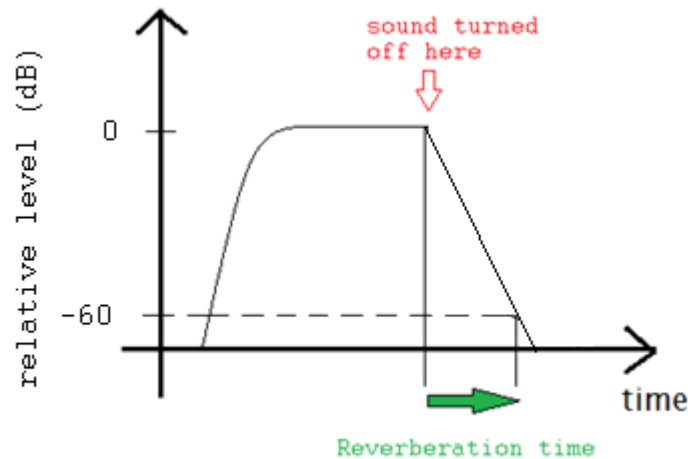
Since reverberation should be the measure of how a sound persists in a room due to reflections, the definition of reverberation should invoke an experiment that can determine that persistence. Here's what we do: turn on a sound in a room, leave it on for a bit and "fill" the room with the sound, then turn it off.



A graph showing what reverberation time represents

The graph just pictured shows what reverberation time means. Once the room has reached what is called a *steady-state intensity*, the sound is shut off and the intensity dies away as sound energy is absorbed into the walls.

By convention, the *reverberation time* is defined as the amount of time it takes the steady-state intensity of a frequency played in a room to die away by 60 dB. This is a drop of a factor of one million in intensity, which is why the 10^{-6} is given as the important “fraction of maximum intensity” shown on the last graph. In dB, the same graph looks as pictured below. The decay process is shown as a straight line because now the y-axis is explicitly a logarithmic scale. (The onset of the sound is not as crucial to this discussion – I don’t need to consider it for our purposes here.)



Another depiction of reverberation time using a decibel scale

Some texts or references use the abbreviation RT60 or RT_{60} to refer to reverberation time, which explicitly links the number 60 to this idea.

A room that is “boomier” or more reverberant will have a longer reverberation time because its reflective walls will allow the persistence of sounds to go on longer. Most concert halls have reverberation times between 1.5 and 2.5 seconds. [This source](#) gives some examples, for example, Carnegie Hall has a reverberation time of 1.8 seconds for a 500 Hz sound, and the Konzerthaus in Berlin has a reverberation time of two seconds. This assumes that the hall is occupied. These halls have larger reverberation times when empty, for reasons we’ll discuss later. On the other hand, a recording studio designed to remove unwanted reverberations might have a reverberation time of half a second or less.

Calculating reverberation time

What does reverberation time depend on?

Reverberation time depends on a few different factors. The architecture of the room is one obvious factor, but I will simplify and quantify this idea somewhat. It depends on the materials of the walls – this should make sense since a room lined in sound-absorbing foam will reverberate less than one with solid brick or concrete walls. It also depends on the frequency of the sound being played. Different materials absorb different frequencies differently.

In summary, reverberation time depends on:

- The volume of the room
- The size AND composition of the walls
- The frequency of the sound being played

Around the turn of the 20th century, Wallace Sabine promoted the field of architectural acoustics with his measurements of reverberation times.

Absorption coefficients and Sabine's reverberation formula

We need to quantify how much a given material will absorb energy. The more a material absorbs, the less it will reflect, and the less reverberation will happen off of such a wall. The *absorption coefficient* is the name we give to the ratio of the energy a surface absorbs divided by the energy coming into it. It is given the Greek letter alpha (α) for a symbol.

$$\alpha = \frac{\text{energy absorbed}}{\text{incident energy}}$$

ACTIVITY DISCUSSION QUESTIONS:

- What are the units of α , if any? How do you know?
- What is the largest possible α ?
- What is the smallest possible α ?



Wallace Sabine ([source](#))

(Answers to questions above:) Absorption coefficient has no units since it's a ratio of energies. The largest possible value is 1 for a surface that absorbs every bit of sound energy it receives, and the smallest value for alpha is 0 for a completely reflective surface.

Sample coefficients

A table of sample sound coefficients can be found [here](#). An excerpt from that table is shown below:

Material	125 Hz	250 Hz	500 Hz	1 kHz	2 kHz	4 kHz
Unglazed brick	0.03	0.03	0.03	0.04	0.05	0.07
Drywall (gypsum) board	0.29	0.10	0.05	0.04	0.07	0.09
3/8" thick plywood	0.28	0.22	0.17	0.09	0.10	0.11
Concrete flooring	0.01	0.01	0.015	0.02	0.02	0.02
Wood flooring	0.15	0.11	0.10	0.07	0.06	0.07
Heavy carpet on concrete	0.02	0.06	0.14	0.37	0.60	0.65
Window glass	0.35	0.25	0.18	0.12	0.07	0.04

Absorption coefficients for various materials and frequencies

Notice here that every material behaves differently. Some materials (like brick) never seem to absorb much energy. Others (like the heavy carpeting) absorb a lot of energy at higher frequencies. Still others (like gypsum and plywood) reflect midrange frequencies better than high or low frequencies. The point is this: you must account for BOTH the room materials and sound frequency when finding reverberation time.

The way we use these coefficients is by defining *absorption area* by multiplying the coefficient for a given wall times the area of that wall:

$$\text{Absorption area} = \text{Area} \times \alpha$$

The unit of absorption area is the same as the unit of area (we use ft² or m², usually), but areas that are specifically absorption areas are called Sabins or metric Sabins in honor of Sabine. To get the total absorption area of a room, add the absorption areas up for all the walls.

You also need the volume of a room to figure out reverberation time. For a rectangular room, this is given by length multiplied by width multiplied by height.

Side note on the use of units: Sabine's reverberation formula is a little unusual in this course since it is unit dependent. By that, I mean there is a constant in the formula that depends on the units of measure you choose. A given room has a fixed volume and walls whose areas don't change, but if you represent those numbers in different units, it can make a difference if the formula depends on the surface area to volume ratio. For example, a cube 1 yard on each side has a volume of 1 cubic yard (1 yd. x 1 yd. x 1 yd.) and a surface area of 6 square yards (6 faces x 1 yd. x 1 yd.). However, if we measure the same cube in feet, it has a volume of 27 cubic feet (3 ft. x 3 ft. x 3 ft.) and a surface area of 54 square feet (6 faces x 3 ft. x 3 ft.). Therefore, if you take (as Sabine does), the quotient V/A, you get 1/6 yd. or 1/2 ft. Of course, these are the same answer, but the NUMBER is different, and we have to account for that. There's a factor of three difference in the numbers here because there are three feet in a yard.

Sabine's reverberation formula:

If the room's dimensions you measure are in feet, the reverberation time RT is given by:

$$RT = \left(0.05 \frac{s}{ft}\right) \times \frac{\text{volume}}{\text{absorption area}}$$

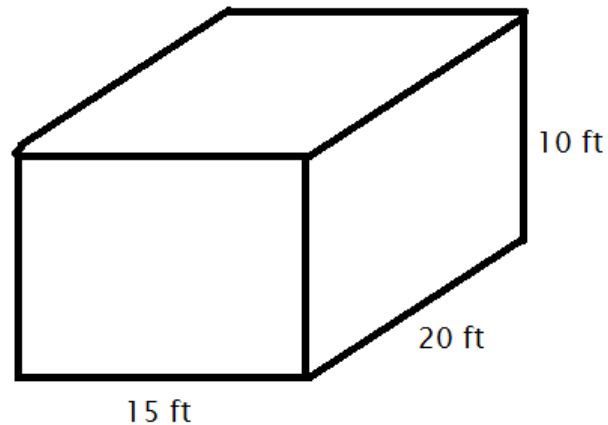
Likewise, if you measure your room in SI units (meters), the RT is given by:

$$RT = \left(0.16 \frac{s}{m}\right) \times \frac{volume}{absorption\ area}$$

Remember that absorption area is more than just the combined area of the walls; you must multiply the area of each wall by the corresponding absorption coefficient. The constants shown have units so the final RT answer has units of seconds, as it should. For more information on the history of architectural acoustics, Wallace Sabine, and his equation, I recommend [this excellent podcast](#) from 99% Invisible.

Example

Suppose you have a 15' x 20' x 10' room. I'll draw it below. The question is: what is the reverberation time for this room given that there's a 500 Hz sound played in it?



We will suppose the floor is hardwood, the back wall is window glass, the other three walls are gypsum drywall, and the ceiling is plywood.

I often find it helpful to figure out the absorption area by putting everything together in a table. I'll do that in a moment. First, though, it's useful to find the volume:

$$V = 15\ ft \times 20\ ft \times 10\ ft = 3000\ ft^3$$

Now, to get the absorption area. I've put in absorption coefficients as needed for the wall materials.

Wall	Area (sq. ft.)	α	Absorption area (sabins) (= Area x α)
Front	15 x 10 = 150	0.05	7.5
Back	15 x 10 = 150	0.18	27
Left	20 x 10 = 200	0.05	10
Right	20 x 10 = 200	0.05	10
Floor	15 x 20 = 300	0.10	30
Ceiling	15 x 20 = 300	0.17	51

If you total the right-hand column to get the total reverberation area, we get 135.5 ft². Therefore,

$$RT = \left(0.05 \frac{s}{ft}\right) \times \frac{3000 ft^3}{135.5 ft^2} = 1.11 s$$

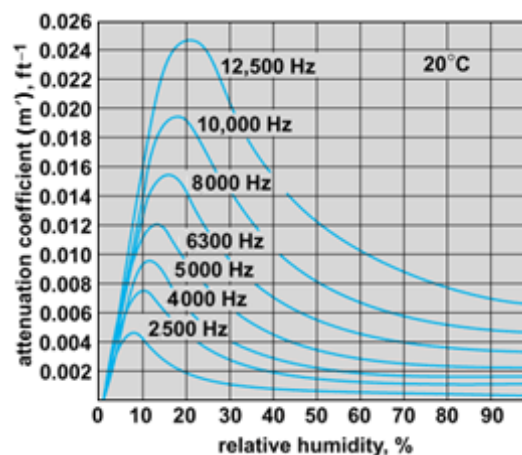
This isn't exactly symphony quality, but it's not an ideal recording studio either. If you wanted this room to be better for most live music, you'd probably replace the back glass wall or the plywood ceiling with more reflective materials to increase the reverberation time. If you wanted it to be better for isolated recording, you might choose instead to add absorbing foam, carpet, or acoustical tile to the more reflective walls.

Supplement: absorption caused by air

For extremely large rooms and high frequencies, the presence of air itself leads to some absorption. Our regular formula (say, for measurements in feet) [must be modified](#):

$$RT = \left(0.05 \frac{s}{ft}\right) \times \frac{volume}{absorption\ area + 4 \times m' \times volume}$$

where m' is an *attenuation coefficient* for air that is frequency and humidity dependent. From the same website:



As you can see, the coefficient is very small for high humidity and low frequency. However, for conditions between 10-30% humidity and frequencies above 500 Hz, the attenuation coefficient starts to become noticeable. Also, you can see from the formula that this air attenuation makes more of a difference in larger rooms.

Homework problems

1. A 100-dB sound source plays for a while and then is turned off. Calculate the intensity of this sound in W/m^2 AFTER an amount of time equal to the reverberation time (RT60) has passed. (hint: after the RT60 has passed, how many dB will this be?)
2. Suppose you had a room that had plywood walls and ceiling (ignore the floor for now). Would it have a longer reverberation time if a 1000 Hz tone were played, or a 500 Hz tone? You shouldn't have to do a lengthy calculation, but carefully explain your reasoning. Check the absorption coefficients in the lecture slides.
3. A 1000 Hz tone is played in a 25 foot by 15 foot room having 12 foot high ceilings. If the floor is concrete ($\alpha = 0.02$), the walls are acoustical tile ($\alpha = 0.75$), and the ceiling is fiberglass ($\alpha = 0.99$), find the reverberation time like we did in class.
4. Suppose you wanted to play orchestral music in the room described in problem #3. (a) What do you need to do to the reverberation time to make it sound better? (b) Give at least two suggestions for material replacements you could make on the walls, ceiling, or floor to change the reverberation time. Some common material α values at 1000 Hz: Plywood 0.09, brick 0.04, curtains 0.32, occupied wooden seats 0.65, wood floor 0.07, drywall 0.04.
5. Let's use the reverb time formula in a different way. Suppose I want to build a room for speeches. Right now, I have an empty room with volume 1500 m^3 and an RT60 of 1.5 s at 250 Hz. (a) What's the absorption area? (b) Suppose I look at the 10 m x 15 m floor and cover the wood-covered concrete ($\alpha = 0.04$) with upholstered, occupied seats ($\alpha = 0.74$). How many metric sabins of absorption area did I gain? (c) What's the new RT60?
6. Consider a room 20 m long, 15 m wide, and 8 m high. The walls are painted concrete block ($\alpha = 0.02$), the ceiling plaster on lath ($\alpha = 0.04$), and the floor carpet on concrete ($\alpha = 0.71$). (a) Neglecting furnishings, calculate the reverberation time for 2000 Hz sound (the frequency that gives the provided absorption coefficients). (b) Suppose we add 200 upholstered seats, half of which are occupied. Use an absorption area of 0.43 metric Sabins when adding a single empty seat and 0.64 metric Sabins for an occupied seat. (c) Air absorption affects the reverberation time in large rooms especially at high frequencies. The new expression for RT (in metric units) accounting for air is

$$RT = 0.16 \frac{V}{A + \alpha_{air}V}$$

where the α for air depends on temperature and relative humidity. Find the new RT if the air's constant is 0.012. (d) Why would the effect of the air be smaller in a small room even if the frequencies played are high?

7. Use the modified reverberation time equation in the previous problem. Suppose you have a room of volume 1000 m^3 and an 8000 Hz sound is played where the air's α constant is 0.136. At this frequency and temperature, we can ignore the absorption by the walls. (a) Find the reverberation time here and (b) If we ignore the walls, would your answer change if the volume were 100 m^3 instead? (Note: a volume of 100 m^3 is likely not big enough where we can truly ignore the walls, but just try it here.)
8. Consider a rectangular music hall 20 m wide, 10 m high, and 30 m long. The floor is asphalt tile ($\alpha = 0.03$ at 500 Hz), the ceiling is covered with acoustical tile ($\alpha = 0.73$), and the walls are gypsum plasterboard ($\alpha = 0.06$). The back wall of the hall has 5 plate glass windows ($\alpha = 0.04$) $2 \text{ m} \times 6 \text{ m}$, and the front end has two open doorways that are $2 \text{ m} \times 3 \text{ m}$ (call the α 1 for these – they don't reflect anything). (a) Find the 500 Hz reverberation time, ignoring the air in the room. (b) A good hall should have a reverberation time of about 1.7 seconds. What should the acoustic absorption area be for this hall? (c) What area of acoustical tile should you place on the walls to get a 1.7 s reverb time?
9. If the speed of sound in a shower stall is 348 m/s , and the dimensions of the stall are $1.5 \text{ m} \times 2.5 \text{ m} \times 3 \text{ m}$. If the walls act as nodes of standing waves, find the fundamental frequencies that act as resonances in each direction. Pretend the three dimensions are string lengths if that helps. (bonus thought) Are any of these singable?
10. A sense of acoustical *intimacy* can be quantified as the time between the arrival of a direct sound and the first reflection of that sound. Intimacy is achieved if the time delay is less than 20 ms . (a) What difference in sound travel distance would create a time difference of 20 ms in room temperature air? (b) Are you likely to get intimate acoustics in a cathedral?